Calculus Conversations
A 1999-2000 Carnegie Scholarship of Teaching & Learning Project in Mathematics

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Problem Background & Goals of Investigation

During the past seven years we have made substantial changes to our three-semester calculus sequence. We use a variety of teaching methods designed to promote self-discovery of mathematical ideas and cooperation with other students. Despite these changes, we continued to see significant evidence that students were unable to apply the methods used in a practiced problem to a new situation. All our efforts to reform calculus had simply increased what learning theorist Robert Sternberg refers to as the "book smart" intelligence of our students. [4] That is, we provided new and creative ways for them to practice their skills, methods and procedures -- all necessary but unfortunately insufficient for fostering a useful understanding of the material. What we failed to provide is an environment that allows students to increase what Sternberg refers to as their "creative intelligence" (i.e., intelligence that allows us to use our knowledge to solve new problems in original ways) or their "street smarts" (i.e., intelligence that allows us to use common sense to find new strategies for solving problems.) [4]

"Prevalent school practices assume, more often than not, that knowledge is individual and self-structured, that concepts are abstract, relatively fixed, and unaffected by the activity through which they are acquired and used, and that Just Plain Folk behavior should be discouraged."[2] In their work, Brown, Collins and Duguid compared problem-solving approaches of Just Plain Folk, students and actual practitioners. They found that Just Plain Folks and practitioners reasoned with casual stories, acting on situations, resolving emergent problems, producing negotiable meaning. On the other hand, students reasoned with laws, acting on symbols, resolving well-defined problems, producing fixed meaning. [2] The goal of our project was to increase students' conceptual understanding of first principles in calculus by creating an activity where students could practice solving problems using a Just Plain Folk approach.

Project Description

Using the tenets of Brown, Collins and Duguid, we set out to more clearly define conditions that might support mathematical development of creative intelligence and street smarts. Anna Graeber, in a talk about what teachers should know about mathematical ways of knowing given at the 1996 International Conference on Mathematical Education, stated that "there is evidence that knowledge is more enduring when it is learned in a meaningful context, through reasoning from relatively primitive concepts, by explaining to others, and by reflecting on one's own knowledge growth."[3]

It is around these simple ideas that we framed our *Calculus Conversation* activity.

In the fall of 1999, we supplemented the activities in our reform calculus courses (lectures, computer labs, out-of-class projects, in-class collaborative group work, and worksheets) with *Calculus Conversations*, a web-based threaded discussion among students. In response to three problems posted to the website by the instructors, students were encouraged to frame thoughtful questions and solutions of their own and to respond to questions and solutions posed by others. The three problems (see Appendix II for a complete listing of the *Calculus Conversation* problems and related exam questions.), posted at regular intervals throughout the semester, focused on ideas central to the major themes of the course. The solutions to the problems required minimal calculation or procedures and could be easily described in narrative form. To further encase the activity in a Just Plain Folk atmosphere, once the problem was posed, instructors took a hands-off approach. The on-line conversation was conducted entirely by the students.

To acquaint the students with *Calculus Conversations* we asked them to post an on-line introduction of themselves to the website. This web posting was followed by an in-class "get acquainted" session where we provided information about what we hoped would be accomplished through their participation in *Calculus Conversations.* The general format for *Calculus Conversations* was to post a question and provide a three- to five-day period for student-to-student interaction. On a set date, the website postings were closed and an in-class conversation among the students in the presence of the instructor took place. One student served as a moderator of the discussion and one student served as a recorder. This in-class component of *Calculus Conversations* allowed students to bring in sketches or graphs of their ideas and gave them an opportunity to explain their solutions in greater depth. On the day following the in-class session, the instructors provided feedback on the students' conversation, which frequently consisted of encasing their ideas in more formal mathematical terms. Our final check on student learning related to the *Calculus Conversation*s question was through an exam question. Participation in *Calculus Conversation*s was both required and graded.

Results

During the semester we collected the following data: actual student contributions to the conversation captured on the website; student *Calculus Conversations* grades on each problem (1 to 4 with 4 being the best score); and student graded responses to exam questions conceptually related to the *Calculus Conversation* problem, but placed in an unfamiliar context (recorded as a % of the total possible).

The quantitative analysis of this project attempts to determine if student performance on conceptual exam questions can be correlated with the quality of their participation in the web-based threaded discussion. The qualitative analysis examines the approaches and language that students use to express their understandings and misunderstanding of mathematical ideas, as well as their abilities to read and respond to the questions and solutions posed by their peers.

Quantitative Results

The primary question of interest was whether the quality of participation in the *Calculus Conversation* activity was related to performance on a conceptually related but contextually varied exam question. Examination of those two variables becomes important to understand the overall analysis. Calculus Conversation responses were coded as high quality (response is clear and well thought out, moves the conversation forward -- 4), average quality (response is understandable, keeps the conversation even -- 3), low quality (response is vague, confuses the conversation -- 2) or did not participate (1). Table 1 shows the overall average performance for students on each of the *Calculus Conversation* problems. The high averages are more indicative of instructor reluctance to discourage participation than to actual contributions by students to the conversation. It is important to mention that there was a noticeable difference in the distribution of grades for the *Calculus Conversation* activity by section (Appendix III, Tables 10 a,b,c) that certainly clouds any conjectures drawn from the quantitative analysis.

TABLE 1

Problem 1	Problem 2	Problem 3
3.65	3.53	3.45

Table 2 shows the average performance of all students on each exam question conceptually related to but contextually varied from the *Calculus Conversation* problems.

TABLE 2

Exam 1 Question Average	Exam 2 Question Average	Exam 3 Question Average
64.48 %	86.97 %	60.03 %

Table 3 shows the average exam question performance for students in each of the *Calculus Conversation* scoring categories. For example, on problem 1 the related exam question average was 43.40 % for students who received a grade of 3 on *the Calculus Conversation* problem. Recall that students could score from 1 to 4 (where 4 was highest) on each *Calculus Conversation* problem. In general one does see a pattern of lower *Calculus Conversation* scores associated with lower grades on the related exam question.

TABLE 3

Problem 1	Exam 1 Question	Problem 2	Exam 2 Question	Problem 3	Exam 3 Question
1	25.00 %	1	83.00 %	1	52.80 %
2	--	2	79.00 %	2	--
3	43.40 %	3	79.80 %	3	53.42 %
4	76.04 %	4	90.56 %	4	64.29 %

A statistical examination of the relationship between the *Calculus Conversation* problem score and the related exam question is significant ($p < .01$). Table 4 shows the results of a Pearson's Product Moment Correlation for each set of relationships.

TABLE 4

Calculus Conversation	Exam 1 Question	Exam 2 Question	Exam 3 Question
Problem 1	.420		
Problem 2		.354	
Problem 3			.503

The results do show that a statistical relationship exists between a student's performance on the *Calculus Conversation* activity and performance on a conceptually related but contextually varied exam question. While we were pleased to see that participation in *Calculus Conversation* correlated with performance on the related exam questions, we do not necessarily conclude that this relationship is causal. Not surprisingly, we also found a statistically significant relationship between students' ACT math scores and their performance on the related exam questions. However, we do note that prior to the insertion of the *Calculus Conversation* activity, average exam scores on questions that were conceptually related to the material being studied, but contextually varied from practiced problems had ranged from 25% to 50 %. This at least leads us to conjecture that the *Calculus Conversation* activity may enhance students' abilities to transfer information learned in one context to a new setting.

We also asked if the relationship between the activity and performance was stronger or weaker in certain populations. Analysis of the data did not allow us to draw many generalizations for performances subdivided by course section, by gender, by high school versus college students, or for first-time calculus students versus those who were repeating it after having already taken the course in high school. For a complete discussion of findings for these populations see Appendix III.

Qualitative Analysis

A qualitative analysis of responses to the web-based *Calculus Conversation* problems was conducted to examine how students addressed and attempted to solve the proposed problems. This type of analysis is done when data are in a narrative form and allowed us to categorize responses into four groups, each representing a skill necessary to correctly analyze the problem. To ensure that the categories developed were clearly defined, comparisons were made between two readers' classification of the student responses. Checking for inter-rater reliability is a key aspect of any analysis of text material. [1] In this case, instances of agreement and disagreement were recorded with the total percent agreements shown in Appendix IV, Tables 11a,b,c. The tables show that the percent agreements do vary, but all are over 50% with some agreements well over 80%.

Responses were categorized as: (P or P-) focus of response was on technical or practical issues such as graphing, collection of data, or modeling and interpretation of data; (C or C-) focus of response was on conceptual issues of the problem through introduction of a new idea, proposed solution, or proposal of a question that spoke to key ideas; (I or I-) intercommunication where responses indicated a student had read and attempted to address an earlier response; (L+ or L-) language and grammar issues that either made a response very clear or alternatively, incomprehensible; and, (Q) response posed a non-rhetorical question. The minus (-) designation was used to indicate that the response confused rather than helped the conversation. Responses could, and often did, receive more than one categorical index.

To illustrate the indexing of responses, in problem 3 the students were asked to analyze the toxic dumping activity of two companies and determine which, if either, was the better environmental citizen. Completing the problem involved mathematically understanding the relationship between the rate at which toxins are dumped into a lake and the total amount of toxin that is accumulated in the lake as a result of the dumping. Most of the responses to this problem (as well as problems 1 and 2) tended to focus on practical issues.

A typical practical response:

> "Since we are dealing with GRAPHS, let me add something.. I think that the most logical way to starting is to define what the graphs will look like. On the dependent axis (the bottom one), will be time. On the independent axis (the left one) will be the TOTAL waste pumped into the river (is it a river? I think so). Because it is TOTAL, the graphs will never have a negative slope. Whatever you do to lessen your output of waste, you are still going to have the amount of waste you had in the past...So the graph will not go down. It is like the AIDS project.. That it was the total number of cases.. the best thing one could hope for was for the graph to level off.. so those are my two cents for now." (JN)

A typical conceptual response:

> "Although both companies reduced the amount of toxins they were dumping by 30%, the nuclear power plant increased the amount of toxins dumped before decreasing it, while Krusty's only decreased the output of toxins.
>
> For example, say that at first they were both dumping at 20 gallons per day (I have no idea if this is realistic or not). Then Krusty Burger worked for 12 months to decrease the amount of toxins dumped to 14 gallons per day (a 30% decrease). The nuclear power plant, however, increased their dumping during that period, and although they to decreased to 14 gallons per day, if you looked at the total amount of toxins dumped theirs would be more than Krusty Burger's. Therefore Krusty Burger is doing a better job at helping the environment." (BA)

A portion of a response that indicates that a student read, understood and was attempting to reply to a particular student:

> "I would like to mainly respond to Jeff's comment that the slope of the Krusty Burger's slope would be the same from t=0 to t =12. It was stated in the problem that the pollution reduction rate was continually reduced until it reached 30%, but that does not mean that the rate of change was constant, and therefore the slopes at all times would not be the same. " (SP)

A response that poses questions for other students:

> "I think we need more info for this question. For example, when the Nuclear Power Plant went off track, where were the two companies in terms with reaching their goal? Were both of them already at 30% or were they some where in the middle? When did the Krusty Burger reach 30%? Did they end up reaching the goal at the same time? I do not totally understand this. If anyone else read something I missed please inform me." (AP)

Listed in tables 5a, 5b and 5c are the distribution of response categories given as a percent of the total number of responses within each section. Evidence in these tables indicates that students clearly prefer practical approaches to problem solving. This supports the claim by Brown, Collins and Duguid that students reason with laws, acting on symbols, looking for fixed meaning. [2] However, arriving at complete solutions to the problems required finding casual, transferable meaning in known laws and definitions. The observed pattern for each problem showed that responses spiraled inward converging toward a correct solution. Early responses on the outer edge of the spiral were almost completely practical in nature. Later responses were more likely to weave together the practical aspects of the solutions with their conceptual counterparts. It is interesting to note that in all three sections on all three problems, the students never completely solved the problem on-line. They hovered just slightly above the solution but were unable to land comfortably on a response that they could build consensus around. Language extremes observed in problem 1 were almost non-existent by problems 2 and 3. It is our conjecture that the students formed language norms in problem 1 that were used in subsequent problems. Interaction among the students in the on-line conversation is clearly section-dependent. The instructor in Section 1 provided a neutral introduction to the activity, made the announcement that a problem had been posted once, and subsequently gave every student full credit if they weighed-in. (Appendix III, Tables 10 a,b,c) Instructors in Sections 2 and 3 provided an enthusiastic introduction to the activity, prompted students regularly to weigh-in early and often, and had a wider distribution of grades. (Appendix III, Tables 10 a,b,c) It is not surprising that interaction among the students appears to depend heavily on both explicit and implicit messages given by the instructor.

TABLE 5a

Problem 1

Category	**Section 1** 25 Students 28 Responses **% of Responses**	**Section 2** 21 Students 39 Responses **% of Responses**	**Section 3** 16 Students 20 Responses **% of Responses**
P or P-	75 %	95 %	85 %
C or C-	68 %	21 %	45 %
I or I-	18 %	64 %	55 %
L+ or L-	18 %	26 %	15 %
Q	11 %	28 %	0 %

TABLE 5b

Problem 2

Category	**Section 1** 25 Students 26 Responses **% of Responses**	**Section 2** 21 Students 24 Responses **% of Responses**	**Section 3** 16 Students 25 Responses **% of Responses**
P or P-	85 %	79 %	60 %
C or C-	15 %	33 %	36 %
I or I-	4 %	58 %	52 %
L+ or L-	0 %	12 %	16 %
Q	4 %	8 %	32 %

TABLE 5c

Problem 3

Category	**Section 1** 25 Students 25 Responses **% of Responses**	**Section 2** 21 Students 27 Responses **% of Responses**	**Section 3** 16 Students 15 Responses **% of Responses**
P or P-	88 %	70 %	67 %
C or C-	68 %	37 %	60 %
I or I-	12 %	56 %	54 %
L+ or L-	0 %	4 %	7 %
Q	8 %	15 %	27 %

Discussion

Following the on-line student-to-student conversation was like watching students formulate mental rough drafts of their understanding and their misunderstanding. According to Graeber, "one needs to understand students' current knowledge if one wants to amend or extend what they know." [3] While admitting that the quantitative and qualitative analysis of this project produced results that were informative, of most interest and value to the instructors was the window that this activity provided into how students think about solutions to problems. We found that students spend an inordinate amount of time mucking around in the details of a problem. This is neither surprising nor bad. What was surprising to the instructors -- and paralyzing for the students -- was their inability to rise above the details of the problems. They also played elaborate word matching games, trying to conjure up definitions and examples of practice problems that contained key terms found in the stated problem. When student JN writes "...the graph will never go down. It is like the AIDS project. That it was the total number of cases," he is attempting to find meaning in the stated problem that asks students to think about total pollutants given information about the rate of pollution from a problem done much earlier in the term that asked students to think about the growth rate of AIDS given the cumulative total number of AIDS cases over a period of time. Making this link helped him to visualize the graph of pollutant totals but he was unable to make the creative leap from seeing the graph to understanding what it tells him about the situation. The most startling observation was the lack of confidence that students have in themselves and each other. In two of the three problems, there was an early, elegantly written solution that was totally ignored by the rest of the class. The author of the response, unsure of her own work, did not bother to weigh in when others in the class posed incorrect solutions. This lack of confidence, both in themselves and their classmates, made forming consensus an impossible task.

The on-line conversation was also a terrific preparatory exercise for the in-class discussions that followed. This was clearly an unanticipated benefit. Because they had been actively engaged in a collaborative effort, the students were prepared for the in-class conversations that followed the on-line activity. Most students, having suffered through the often confusing thoughts of their classmates, wanted clarification. Our supposition was that motivating them to want a clear resolution of the problems would enhance their understanding of the concepts. The result of their performances on conceptual exam questions (see Table 2) clearly indicates that this was not always the case.

Implementing a web-based student-to-student discussion is a very economic way to learn what students know and what they don't know about a particular mathematical idea. Because the forum is so public, most students feel pressured to think carefully before posting their ideas. For the instructor, watching the conversation unfold provides interesting moments of reflection about students' understandings and misunderstandings. Grading the student responses to the *Calculus Conversation* problem was very straight forward and took very little time. Finally, using web-based discussions in this manner makes capturing student work for study at a later date effortless.

If the enterprise we call the scholarship of teaching and learning is, at its core, about creating and studying strategies that provide insight to student learning, then I believe this project is on the right track, but with many miles to go. In this first attempt to provide students with an opportunity to deepen their understanding of calculus, the instructors were the ones whose understanding was enhanced. Following a year of intensive work on this problem, we are now poised to improve the *Calculus Conversation* activity, to frame better questions (Can the nature of a problem prompt students to be more practical or more conceptual? Do students get better at solving the problems? Do they get better at expressing their ideas mathematically?) and to design a better study. In partial answer to Lee Shulman's question, this is a case of an attempt to improve students' conceptual understanding of fundamental ideas in calculus that resulted in improved teacher understanding of what students know and don't know about key concepts in calculus.

Sources Cited

[1] Bordens, K.S. and Abbott, B. B. Research Design and Methods, Mayfield Publishing Company, Mountain View, California., 1996.

[2] Brown, J.S., Collins, A., Duguid, P *Situated Cognition and the Culture of Learning*, "Educational Researcher", Jan-Feb, 1989, pgs 32-42.

[3] Graeber, A. as cited in Seldon, A, Seldon J., http://www.maa.org/t_and_l/sampler/rs_1.html , *Of What Does Mathematical Knowledge Consist*, 1996

[4] Sternberg, R.J., Pathways to Psychology, pgs 266-268, Harcourt Brace & Co., 1997.

For more information on this project e-mail Anita Salem at anita.salem@rockhurst.edu.
For information about other mathematics projects of a similar nature go to the Teaching & Learning MAA site: http://www.maa.org/t_and_l/sampler/research_sampler.html

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Appendix I - Website Access:

To access the Calculus I (Differential Calculus) website go to: http://webct.rockhurst.edu, Course Listings, Archive & Development, Calculus Conversations. Guest user ID ia 2000 and password is "password". Once you are in the website, select *Bulletin Board* and you may look at the following forums: MT180 A archive, MT180 B archive and MT180 C archive

Appendix II – General Instructions, Complete Problem Listing & Related Exam Questions

General Instructions to the Students

Remember that our purpose in thinking about this question is to help us (as a calculus learning community) better understand the essential concepts in calculus. Surprisingly, there are not many essential ideas in calculus so it is important that we all construct correct and meaningful understandings of what they are and how they work. Central to Calculus I is the notion of a function. This first question asks you to think about some particular functions.

Our goal, as a class, is to arrive at a correct and thoroughly explained answer to the posed question. Individually your goal should be to move the conversation, and our collective understanding forward. Your response to this question may take a variety of forms. You may want to suggest an answer to the question. You may want to offer a clarification of the meaning of a particular term or phrase. You may want to pose a thoughtful question whose answer might help you think about the original question in a different way. You may want to reply (with a question, correction, clarification or elaboration) to a response from another classmate. You may want to pose an interesting problem that bears some relationship to the original question. After reading the responses of your classmates, you may want to formulate a succinct synopsis. Regardless of the form it takes, your response should be clearly explained. Remember, you are writing to your classmates and you want your response to further their understanding.

In order to keep the conversation organized, we ask that you be very careful about threading your responses appropriately. If you are responding to the original question, click on instructor and reply. If, however, you are responding to a particular student, click on that student's response and reply. NEVER SELECT THE COMPOSE BUTTON IN THE MENU.

You are required to respond and your responses will be graded as high quality (response is clear & well thought out, moves the conversation forward - 20/20), average quality (response is understandable, keeps the conversation even - 15/20), low quality (response is vague, confuses the conversation - 10/20) or did not participate (0/20). High quality responses come in a variety of forms. Of course, we would like to see you answer the original question. However, we are also looking for good questions that emanate from the original question. You are welcome to respond more than once and your grade will be determined by the total contribution that you make to the final solution of the question. It goes without saying that all responses should be respectful of the ideas of others.

Instructions for Student Introductions

This website is shared by students in Calculus I (Sections A, B, C), Calculus II and Calculus III. Its primary purpose is to allow you to become involved in the community of calculus learners. Calculus Conversations is a place where you can ask questions or pose new problems. It is also a place where you can help answer questions and problems posed by your classmates. Together, we will work on categorizing the questions and problems, restating them in clear mathematical terms, and we will suggest strategies for finding solutions. In a small, but important way we will be working together towards a deeper understanding of how mathematical ideas can help us ask and answer interesting and important questions. To help us get to know each other better, we ask that you each reply to this message by posting a brief introduction of yourself. Let us know a little about why you are taking calculus, and share with us a few personal facts: where are you from? any hobbies? what do you want to be when you grow up? etc. Postings are DUE by class time Friday, September 10th.

Calculus Conversation Problem 1 (Author: Tom Banchof)

Has there ever been a time in your life when your height in inches has equaled your weight in pounds? Mathematically explain your answer.

Exam Question Related to Problem 1

Let $f(x)$ and $g(x)$ be functions defined on the interval $a <= x <=$ b. Additionally suppose that $f(a) < g(b)$. Sketch the graphs of $f(x)$ and $g(x)$ on the interval $[a,b]$ so that at all values of c between a and b, $f(c)$ is not equal to $g(c)$. (i.e., f and g do not cross). What condition must hold in order to guarantee that $f(x) = g(x)$ for some value of x between a and b?

Calculus Conversation Problem 2

In the Jesuit spirit of becoming men and women for others, you have decided to take part in a 5-mile charity walk. You are told that refreshments will be handed out to all volunteer walkers as they pass the 2.5-mile marker. You decide to walk at a constant speed of 3 miles per hour, and to pass the time you also decide to track your distance from the 2.5-mile marker during the entire walk. (Note that whether the 2.5 mile marker is ahead of you or behind you, your distance from the marker is to be considered non-negative.) At the INSTANT you pass by the 2.5-mile marker, what can you say about the rate at which your distance from the marker is changing relative to time?

Exam Question Related to Problem 2

Suppose that you are given the formula for a function $f(t)$ and you have no idea how to calculate the derivative, $f'(t)$. Describe how you could obtain a good approximation for the instantaneous rate of change of $f(t)$ at $t=5$. Assume that you have available some technology which gives you full graphing and computing capabilities but not the ability to calculate derivatives. Please describe the process completely and in adequate detail.

Calculus Conversation Problem 3

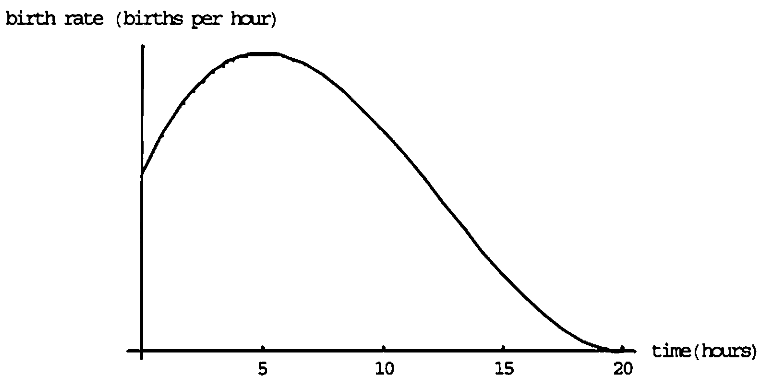
Two industries, the Springfield Nuclear Power Plant and the Krusty Brand Burger Company, are being charged by the Environmental Protection Agency (EPA) with dumping unacceptable levels of toxic pollutants into Springfield Lake. Both industries are currently dumping toxic pollutants into the lake at the same rate. In order to meet EPA compliance standards, over the next twelve months both industries must reduce the rate at which they are dumping toxic pollutants in the lake by 30%. The presidents of Springfield Nuclear Power Plant and Krusty Brand Burger hire a single engineering firm to develop a compliance plan acceptable to both companies. The plan maps out a specific schedule of continual reduction in the toxic dumping rate over the entire twelvemonth period ending in the required 30% reduction in the dumping rate as specified by the EPA. The Krusty Brand Burger Company follows the plan as outlined by the engineering firm. The Springfield Nuclear Power Plant manages to stay on plan for the first three months. Because of equipment breakdowns and delays in getting replacement parts, during the next three months the reduction rate achieved during the first three months is reversed, and at the end of the first six-months, the Springfield Nuclear Power Plant dumping rate is back to where it was in the beginning. Once the equipment is repaired, the industry uses extra resources to continually reduce the dumping rate, and over the remaining six months the company manages to meet the30% reduction in the dumping rate set by the EPA. COMPLETELY DESCRIBE the graphs of each company's individual contribution to the total level of toxic pollutants in the lake as a function of time over the twelve-month EPA observation. Has either industry been a better environmental citizen? Mathematically explain your answer.

Exam Question Related to Problem 3

Consider a bacteria population whose birth rate changes in the following ways during a twenty-hour period:

- The birth remains nonnegative (i.e., positive or zero) during the entire twenty hour period.
- The birth rate begins the twenty-hour period at a positive value.
- During the first five hours, the birth rate continually increases and reaches a maximum value at the end of five hours.
- During the next fifteen hours, the birth rate continually decreases, reaching a level of zero right at the end of the twenty-hour period.

The graph of the birthrate as a function of time in hours is shown below for this twenty hour period. The birth rate is in given in births per hour.



Explain what the birth rate curve tells you about the total number of individuals born (during the twenty-hour period) by time t. Does the graph of the total number born by time t have an inflection point during the twenty-hour period? If so, at what time (within the twenty-hour period) does it occur? Sketch the graph of the total number of individuals born (during the twenty-hour period) by time t.

Appendix III - Quantitative Analysis for Specific Populations

Tables 7 a, b, and c show the average scores of students on the Exam Question by activity score and by gender.

TABLE 7a

Calculus Conversation Grade on Problem 1		Mean	N	Standard Deviation
	Male	0.00	1	---
1	Female	33.33	3	28.87
	Male	---	---	---
2	Female	---	---	---
	Male	52.83	6	52.04
3	Female	29.25	4	15.75
	Male	59.61	26	38.01
4	Female	78.17	36	25.97

TABLE 7b

Calculus Conversation Grade on Problem 2		Mean	N	Standard Deviation
	Male	83.00	2	.00
1	Female	--	--	--
	Male	--	--	--
2	Female	79.00	4	8.00
	Male	81.22	9	5.33
3	Female	77.67	6	17.11
	Male	92.13	15	10.71
4	Female	89.65	26	11.62

TABLE 7c

Calculus Conversation Grade on Problem 3		Mean	N	Standard Deviation
	Male	53.50	4	37.12
1	Female	50.00	1	---
	Male	--	--	--
2	Female	--	--	--
	Male	57.87	8	34.69
3	Female	50.18	11	29.54
	Male	62.71	14	22.87
4	Female	65.21	24	25.27

Tables 8 a, b, and c show the average score of students on the Exam Question by activity score and their status as a High School or College student.

TABLE 8a

Calculus Conversation Grade on Problem 1		Mean	N	Standard Deviation
	College	.00	2	.00
1	High School	50.00	2	.00
	College	--	--	--
2	High School	--	--	--
	College	43.40	10	41.66
3	High School	--	--	--
	College	73.49	39	34.12
4	High School	87.11	9	19.95

TABLE 8b

Calculus Conversation Grade on Problem 2		Mean	N	Standard Deviation
	College	83.00	2	.00
1	High School	--	--	--
	College	79.00	4	8.00
2	High School	--	--	--
	College	79.8	15	11.14
3	High School	--	--	--
	College	90.47	30	11.32
4	High School	90.82	11	11.48

TABLE 8c

Calculus Conversation Grade on Problem 3		Mean	N	Standard Deviation
	College	52.80	5	32.18
1	High School	--	--	--
	College	--	--	--
2	High School	--	--	--
	College	53.42	19	31.11
3	High School	--	--	--
	College	60.81	27	25.10
4	High School	72.82	11	20.09

Tables 9 a, b, and c show the average score of students on the Exam Question by activity score and their status as a first time Calculus student or as a repeating student. A repeating student is one who has typically taken Calculus in High School and is repeating the experience at the college level.

TABLE 9a

Calculus Conversation Grade on Problem 1		Mean	N	Standard Deviation
1	First-time	33.33	3	28.87
	Repeater	.00	1	--
2	First-time	--	--	--
	Repeater	--	--	--
3	First-time	52.12	8	42.14
	Repeater	8.5	2	12.02
4	First-time	76.55	27	28.93
	Repeater	73.38	21	36.75

TABLE 9b

Calculus Conversation Grade on Problem 2		Mean	N	Standard Deviation
1	First-time	83.00	1	--
	Repeater	83.00	1	--
2	First-time	79.00	4	8.00
	Repeater	--	--	--
3	First-time	77.55	9	11.61
	Repeater	83.17	6	10.44
4	First-time	88.79	24	11.69
	Repeater	93.06	17	10.35

TABLE 9c

Calculus Conversation Grade on Problem 1		Mean	N	Standard Deviation
1	First-time	53.50	4	37.11
	Repeater	50.00	1	--
2	First-time	--	--	--
	Repeater	--	--	--
3	First-time	51.07	13	30.37
	Repeater	58.50	6	35.01
4	First-time	59.67	21	22.72
	Repeater	70.00	17	25.27

Tables 10 a, b, and c show the distribution of *Calculus Conversation* scores by section.

TABLE 10a

Coded *Calculus Conversation* Activity Score for Question 1

Section	Problem Grade = 1	Problem Grade = 2	Problem Grade = 3	Problem Grade = 4
1	4			21
2			3	18
3			7	9

TABLE 10b

Coded *Calculus Conversation* Activity Score for Question 2

Section	Problem Grade = 1	Problem Grade = 2	Problem Grade = 3	Problem Grade = 4
1				25
2			6	13
3		4	9	3

TABLE 10c

Coded Calculus Conversation Activity Score for Question 3

Section	Problem Grade = 1	Problem Grade = 2	Problem Grade = 3	Problem Grade = 4
1	3			22
2			8	13
3	2		11	3

One can see that the distribution of scores does vary by section, especially in the percent of the class receiving the highest score for the *Calculus Conversation* activity.

Appendix IV - Role of Reliability in Qualitative Analysis

Tables 11a,b,c show the content analysis of the qualitative data indicating inter-rater reliabilites. (P = practical issue, C = conceptual issue, L = language skill I = intercommunication , Q = question)

TABLE 11a

Calculus Conversation Problem 1

	P	C	L	I	Q
CLASS A (PS)	77%	77	82	64	90
CLASS B (AS)	65	65	90	80	90

CLASS C (JK)	57	75	86	96	93
OVERALL	68	74	85	78	91

TABLE 11b
Calculus Conversation Problem 2

	P	C	L	I	Q
CLASS A (PS)	67%	58	92	75	96
CLASS B (AS)	80	76	88	84	84
CLASS C (JK)	73	88	100	100	96
OVERALL	73	75	93	87	95

TABLE 11c
Calculus Conversation Problem 3

	P	C	L	I	Q
CLASS A (PS)	52%	63	96	81	100
CLASS B (AS)	67	60	93	93	93
CLASS C (JK)	68	52	100	100	96
OVERALL	61	58	97	91	97